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Engineering Notes

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Energy Rate Feedback for Libration Control of Electrodynamic Tethers

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Introduction

ELECTRODYNAMIC tethers are expected to be a viable and efficient means for generating power or controlling the orbit of future spacecraft.[1–5] For cases where the tether current is not explicitly controlled for orbital maneuvering, then the long-term dynamic behavior of the tether librations must be controlled. The control of electrodynamic tethers on inclined orbits is important because of the inherent instability provided by the electric current.[6] It has been conclusively established that, even for a tether modeled as an inelastic rod, the variation in electrodynamic forces over an orbit causes an influx of energy into the libration dynamics. Although, for the most part, the instability is slow to develop, it must be controlled for long-term operations. There exists a unique libration cycle, however, for which the net energy input per orbit is zero. Such a libration cycle is a periodic solution to the equations of motion for the system.[7] The periodic solution is unstable, with instability growing with the electric current.[6] Periodic solutions have also been determined for tethers including the lateral dynamics.[8] The instability in the lateral dynamics grows at a faster rate than the librational instability. The effect of orbit eccentricity was shown by Pelaez and Andres[9] to amplify the unstable nature of the dynamics. Corsi and Iess[10] considered a simple control scheme to stabilize the tether librations for deorbiting applications. Hoyt[11] has described schemes for stabilizing flexible electrodynamic tethers during deorbit by periodically measuring the motion of several points along the tether to estimate the oscillation energy, and adjusting the control input accordingly. The application of electromagnetic forces for directly controlling the librational motion has also been considered.[12–15] Control of the unstable skip-rope motion during retrieval was shown to be effectively suppressed by Williams et al.[15] if a movable attachment point is used to damp the lateral oscillations of the electrodynamic tether.

In Ref. 16, Pelaez and Lorenzini addressed the issue of controlling the librations of an electrodynamic tether around the periodic solution. Their approach consisted of two linear feedback control schemes. The first was simple proportional feedback for the in- and out-of-plane libration rates, and the second was a delayed-feedback version of the same controller. However, the implementations of these controllers were assumed to be independent, where it was suggested that control could be achieved using movement of the tether attachment point. This might not be as efficient or effective as utilizing the electric current itself to provide the feedback. The purpose of this Note is to explore the possibility of controlling the librations of an electrodynamic tether around the periodic solution using modulations in the electric current. The control law is derived as a nonlinear feedback controller, whose stability is examined using Floquet's theory for different current levels and control gains.

System Model

In this work, the tether model adopted by Pelaez and Lorenzini[16] is used. In this model, the mother satellite is assumed to remain in a circular orbit, and the tether is modeled by a rigid rod undergoing forced librations caused by the electric current. For more generality, the nondimensional parameter $\varepsilon = f_e I \mu_m / \mu(m_s + m_t/3)$ is used, where f_e is a parameter governing the distribution of electric current along the tether, I is the current, μ_m is the magnetic field strength, μ is the gravitational constant of the Earth, m_s is the subsatellite mass, and m_t is the tether mass. Essentially, the parameter ε governs the ratio of torque caused by electrodynamics to the torque as a result of gravitational forces.

Under these assumptions, the kinetic and potential energy of the tether are given by

$$\mathcal{T} = \tfrac{1}{2}(m_s + m_t/3)\omega^2 l^2[\phi'^2 + (\theta' + 1)^2 \cos^2\phi] \tag{1}$$

$$\mathcal{V} = -\tfrac{3}{2}\omega^2(m_s + m_t/3)l^2 \cos^2\theta \cos^2\phi \tag{2}$$

where $()' = \mathrm{d}/\mathrm{d}(\omega t)$ is the nondimensional time derivative. The application of Lagrange's equations leads to the equations of motion

$$\theta'' = 2(\theta' + 1)\phi' \tan\phi - 3\sin\theta\cos\theta - \varepsilon[\sin i \tan\phi(2\sin\nu\cos\theta - \cos\nu\sin\theta) + \cos i] \tag{3}$$

$$\phi'' = -\sin\phi\cos\phi[(\theta' + 1)^2 + 3\cos^2\theta] + \varepsilon(2\sin\nu\sin\theta + \cos\nu\cos\theta)\sin i \tag{4}$$

The generalized forces appearing in Eqs. (3) and (4) are derived elsewhere.[16] Equations (3) and (4) have periodic solutions that can be obtained by Poincare's method of the continuation of periodic orbits.

Control Law

In the absence of electrodynamic forces, the Hamiltonian of the system is constant and is defined according to

$$\mathcal{H} = \frac{\partial\mathcal{T}}{\partial\theta'}\theta' + \frac{\partial\mathcal{T}}{\partial\phi'}\phi' - \mathcal{T} + \mathcal{V} \tag{5}$$

In nondimensional form, this is evaluated as

$$\mathcal{H} = \tfrac{1}{2}\phi'^2 + \tfrac{1}{2}\theta'^2\cos^2\phi - \tfrac{1}{2}(\cos^2\phi + 3\cos^2\theta\cos^2\phi) \tag{6}$$

This can be interpreted as the energy of the librational motion relative to the mother satellite. For an electrodynamic tether, energy is continuously being pumped into or removed from the system as a

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result of the electric current. The time derivative of Eq. (6) can be determined using Eqs. (3) and (4) as

$$\begin{aligned}\mathcal{H}' &= Q_\theta \theta' \cos^2\phi + Q_\phi \phi' \\ &= \varepsilon\{\phi'(2\sin\nu\sin\theta + \cos\nu\cos\theta)\sin i \\ &\quad - [\sin i \tan\phi(2\sin\nu\cos\theta - \cos\nu\sin\theta) + \cos i]\theta'\cos^2\phi\} \end{aligned} \tag{7}$$

For a periodic orbit, the net energy input into the system is zero. Hence, we can define the reference energy flux according to

$$\mathcal{H}'_{\mathrm{ref}} = \varepsilon_{\mathrm{ref}} E_{\mathrm{ref}} \tag{8}$$

where

$$\begin{aligned}E_{\mathrm{ref}} &= \phi'_{\mathrm{ref}}(2\sin\nu\sin\theta_{\mathrm{ref}} + \cos\nu\cos\theta_{\mathrm{ref}})\sin i \\ &\quad - [\sin i\tan\phi_{\mathrm{ref}}(2\sin\nu\cos\theta_{\mathrm{ref}} - \cos\nu\sin\theta_{\mathrm{ref}}) \\ &\quad + \cos i]\theta'_{\mathrm{ref}}\cos^2\phi_{\mathrm{ref}}\end{aligned} \tag{9}$$

A feedback controller is proposed of the following form

$$\varepsilon = \varepsilon_{\mathrm{ref}}[1 - k(E - E_{\mathrm{ref}})E_{\mathrm{ref}}\,\mathrm{sign}\,\varepsilon_{\mathrm{ref}}] \tag{10}$$

where k is a feedback gain. The effectiveness and stability of this control law is analyzed using Floquet's theory.

Numerical Computations

The reference periodic trajectories in this Note are determined using the method of continuation of periodic trajectories. A predictor-corrector strategy developed in Ref. 17 is used. The stability of the closed-loop solution can be assessed by using Floquet's theory for periodic systems. The stability of the solution is governed by the magnitude of the eigenvalues λ of the monodromy matrix. The monodromy matrix is formed by integrating the perturbed linear equations of motion in matrix form with the initial conditions equal to the identity matrix. The eigenvalues are calculated from the solution of the differential equations at $\nu = 2\pi$. For this system, we have four eigenvalues. If the modulus of all eigenvalues are less than one, then the system is stable. However, if one of the eigenvalues has a modulus greater than one, then the system is unstable.

The stability properties of the control law were evaluated for $0 < \varepsilon \leq 1.5$ and for orbit inclinations from 5 to 80 deg in increments of 5 deg. Values of k from 0 to 1000 were compared. In this range, for certain configurations and nominal current levels there are no stable solutions. This is particularly true for large inclinations and large nominal ε. For the sake of brevity, only a case study of the orbit inclination 45 deg is performed in this Note. Figure 1 shows the results of a stability analysis of the closed-loop system for varying levels of the control gain k. For this orbit inclination, the control gain is sensitive to both low values and large values, depending on the nominal value of ε. For example, for very small values of ε there is virtually no upper limit on the control gain for stability. However, as the value of ε is increased, there is a lower threshold for the control gain above which the closed-loop solution is stable, but below which the system is unstable. A close-up of the curve is shown in Fig. 1b. As an example, for $\varepsilon = 1.5$ the control gain $k > 0.3$ for the system to be stable. However, there is also an upper bound on the control gain, shown in Fig. 1a. The upper bound is both practically and dynamically desirable because too high a control gain could cause the tether to "flip" over or commence rotation. In a practical sense, the level of current is restricted by physically important parameters such as the ionospheric density of the ambient environment, temperature of the tether, and so on. Hence, the electric current could not be forced at such large levels that would likely correspond with such high values of k. Therefore, the lower bound on k is of more practical importance than the upper bound. Figure 2 illustrates the typical behavior of the closed-loop system for high nominal current levels. For small values of the control gain, there are two complex conjugate eigenvalues of the monodromy matrix, one of which has a moduli greater than one. At a critical value of $k = 0.2988$, the system becomes stable, and the moduli of the eigenvalues decrease. At $k = 1.84$, one of the complex eigenvalues splits into two real eigenvalues. A similar branching occurs at $k = 2.2$. However, one of the branches from $k = 1.84$ eventually becomes unstable at $k = 9.86$, which represents the upper bound on the control gain for this nominal current level.

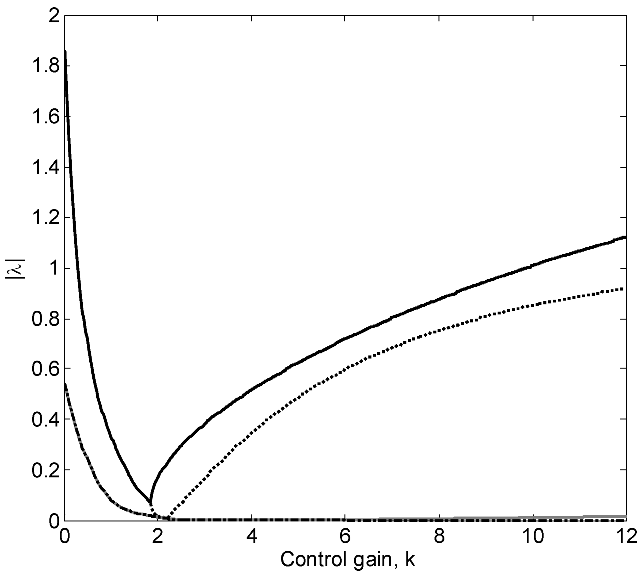

Fig. 2 Moduli of eigenvalues of the monodromy matrix vs the control gain k for 45-deg inclination and $\varepsilon = 1.5$.

To assess the closed-loop behavior of the nonlinear system, numerical simulations have been performed starting with the initial conditions perturbed randomly from the periodic solution. The orbit inclination is 45 deg, and the nominal value of ε is 1.5. For the sake of brevity, only one example simulation is shown in Fig. 3. The initial perturbations to the initial conditions for this case were

$$\delta\theta = -12\,\mathrm{deg}, \qquad \delta\theta' = 0.176$$
$$\delta\phi = 1.78\,\mathrm{deg}, \qquad \delta\phi' = -0.09 \tag{11}$$

The plot in Fig. 3, showing the trajectory for 20 orbital revolutions together with the reference trajectory, illustrates the excellent closed-loop behavior of the system even for this large perturbation from the periodic solution. The closed-loop trajectory approaches the periodic solution in only a few orbits and eventually tracks the periodic solution with negligible error. Figure 4 shows the corresponding nondimensional control parameter. This shows that the control corrections near the beginning of the simulation are relatively large, but

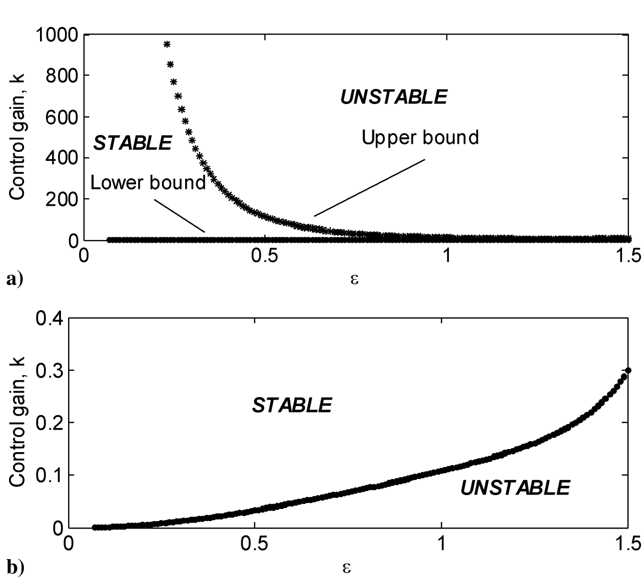

Fig. 1 Stability regions for nonlinear control law for 45-deg inclination: a) complete region showing upper and lower bounds and b) close-up of lower bound.

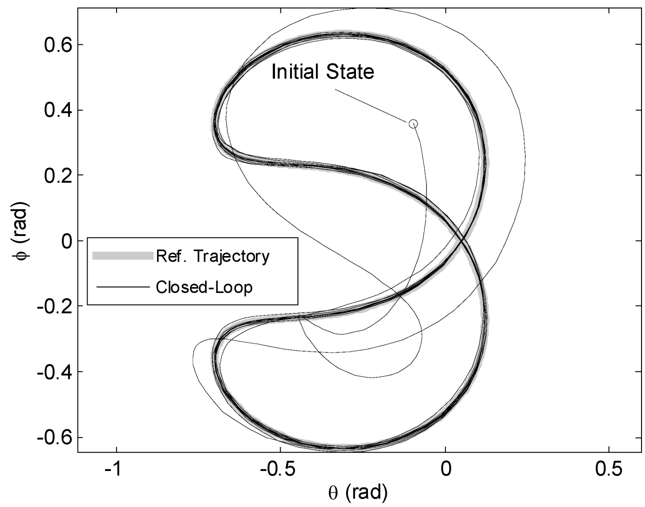

Fig. 3 Closed-loop simulation of electrodynamic tether with large initial perturbation, inclination 45 deg (20 orbits).

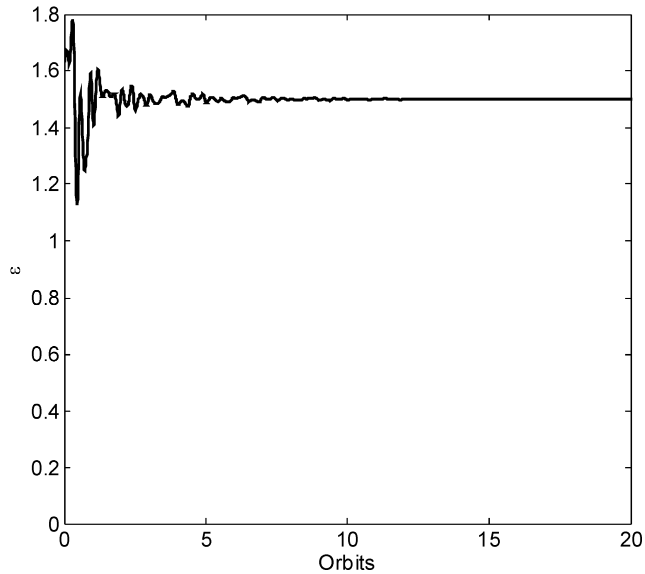

Fig. 4 Control parameter vs orbits, inclination 45 deg.

as the system begins to track the periodic solution with greater precision the control requirements are very modest. Thus, tracking the periodic reference trajectories using only electric current variations superimposed on the nominal mission current might be a viable means for controlling the librational dynamics of electrodynamic tethers.

Conclusions

A nonlinear control law for tracking periodic reference trajectories for electrodynamic tethers on inclined orbits has been developed. The control law uses feedback of the difference in the librational energy rate of the real system from the reference periodic trajectory. The stability of the closed-loop system was analyzed using Floquet's theory, which shows two unstable regions for an orbit inclination of 45 deg. One unstable region occurs for very low values of the feedback gain parameter, and another occurs for very large values of the feedback gain parameter. In a practical sense, only the lower unstable region is of interest. Numerical simulations of the controlled nonlinear system demonstrate the ability of the control law to track the reference periodic trajectories.

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